Skalarny i wektorowy potencjały pola

$$\boxed{\begin{aligned}\vec{E} &= -\operatorname{grad}\varphi - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} &= \operatorname{rot}\vec{A}\end{aligned}}$$

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}$$

$$B = \nabla \times A$$

$$\Rightarrow \quad \operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

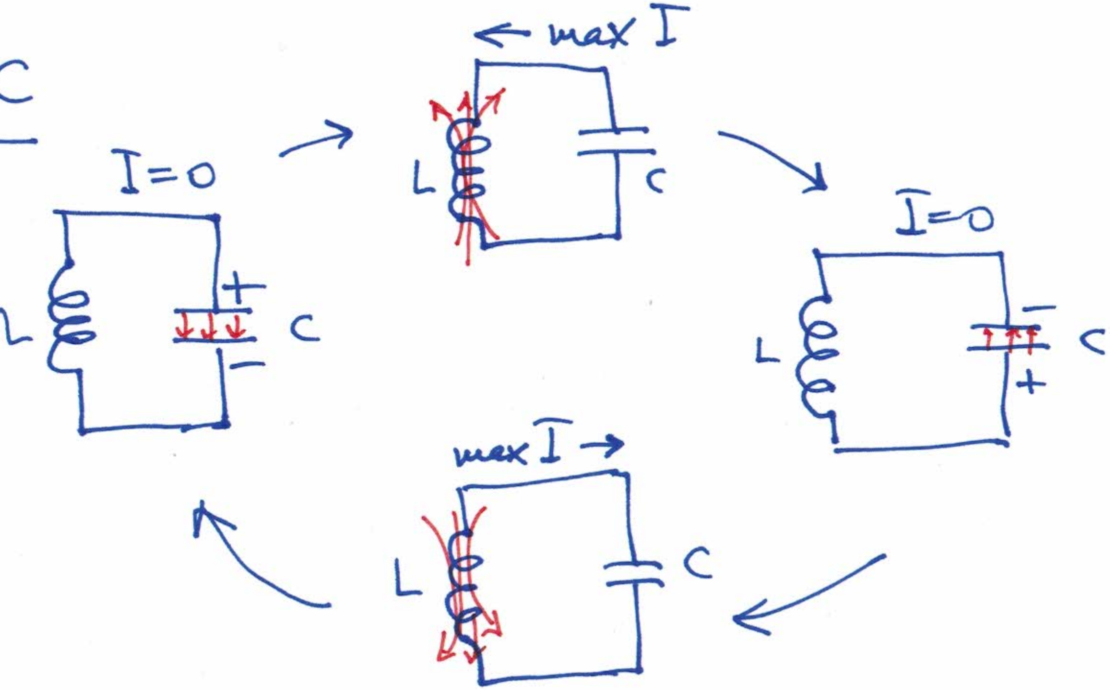
$$\operatorname{div}\vec{B} = 0$$

$$\operatorname{rot}\operatorname{grad}\varphi = 0$$

$$\operatorname{div}\operatorname{rot}\vec{A} = 0$$

Drgania elektromagnetyczne

Obwód LC



$$E_E = \frac{q^2}{2C}$$

$$E_B = \frac{LI^2}{2}$$

W mechanice:

$$E_k = \frac{mv^2}{2}, \quad E_p = \frac{1}{2}kx^2$$

$$E = E_k + E_p$$

$$E = E_B + E_E = \frac{LI^2}{2} + \frac{q^2}{2C}$$

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{LI^2}{2} + \frac{q^2}{2C}\right) = LI\frac{dI}{dt} + \frac{q}{C}\frac{dq}{dt} = 0$$

$$I = \frac{dq}{dt}$$

$$\frac{dI}{dt} = \frac{d^2q}{dt^2}$$

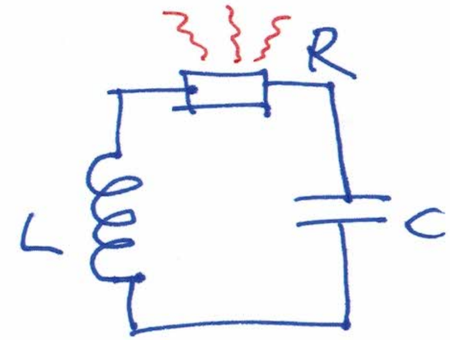
$$\Rightarrow \quad L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

$$\boxed{q = q_{max}\cos(\omega t + \phi)}$$

$$\boxed{I = \frac{dq}{dt} = -\omega q_{max}\sin(\omega t + \phi)}$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

Drgania tłumione w obwodzie RLC



$$E = E_B + E_E = \frac{LI^2}{2} + \frac{q^2}{2C}$$

$$\frac{dE}{dt} = -I^2 R \neq 0$$

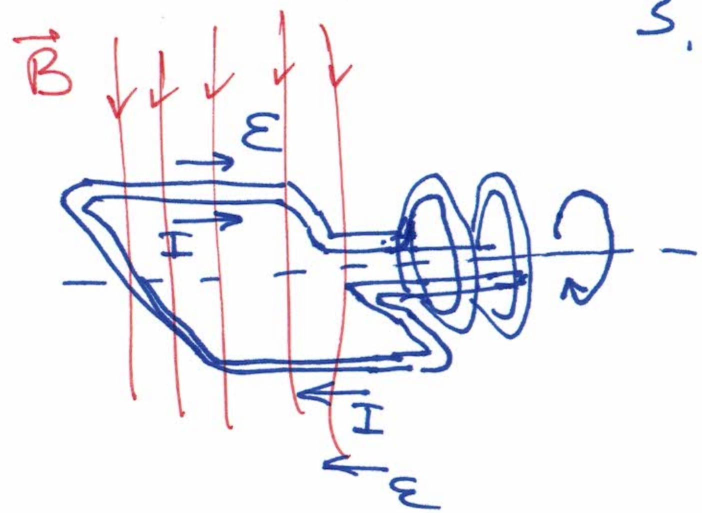
$$\frac{dE}{dt} = LI\frac{dI}{dt} + \frac{q}{C}\frac{dq}{dt} = -I^2 R$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$

$$\Rightarrow \boxed{q = q_{max} e^{-Rt/2L} \cos(\omega' t + \phi)}$$

$$\omega' = \sqrt{\omega^2 - (R/2L)^2}, \quad \omega = \frac{1}{\sqrt{LC}}$$

Prąd zmienny



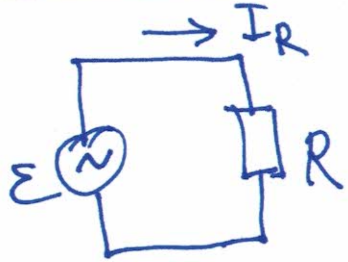
Prądnica prądu zmiennego

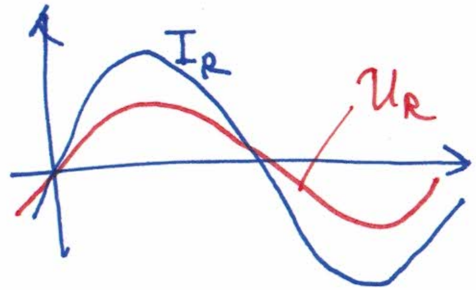
$$\mathcal{E} = \mathcal{E}_{max} \sin \omega_w t$$

ω_w - częstość kołowa SEM

$$I = I_{max} \sin(\omega_w t - \phi)$$

Obwód z opornikiem





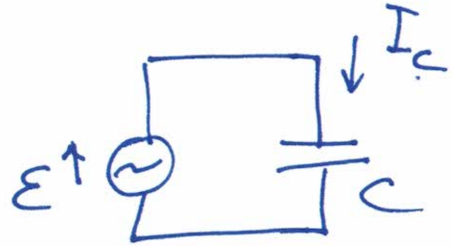
$$\mathcal{E} - U_R = 0 \quad \Rightarrow \quad U_R = \mathcal{E}_{max} \sin \omega_w t = U_{R\,max} \sin \omega_w t$$

$$I_R = \frac{U_R}{R} = \frac{U_{R\,max}}{R} \sin \omega_w t = I_{R\,max} \sin(\omega_w t - \phi)$$

$$U_{R\,max} = I_{R\,max} R$$

$$\boxed{\phi = 0}$$

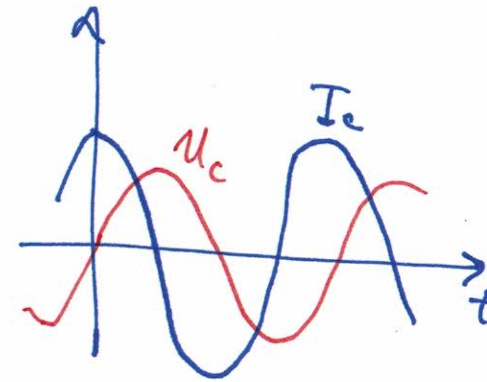
Obciążenie pojemnościowe



$$u_c = U_{c\,max} \sin \omega_w t$$

$$q_c = C u_c = C U_{c\,max} \sin \omega_w t$$

$$I_c = \frac{dq_c}{dt} = \omega_w C U_{c\,max} \cos \omega_w t$$



Wprowadzimy reaktancję pojemnościową kondensatora

$$X_c = \frac{1}{\omega_w C}$$

$$I_c = \frac{U_{c\,max}}{X_c} \sin(\omega_w t + 90^\circ)$$

$$\cos \omega_w t = \sin(\omega_w t + 90^\circ)$$

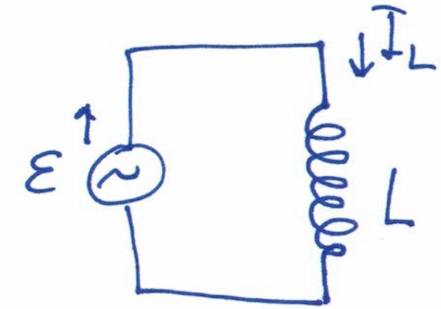
$$\boxed{U_{c\,max} = I_{c\,max} X_c}$$

$$\boxed{I_c = I_{c\,max} \sin(\omega_w t - \phi)}$$

$$\boxed{\phi = -90^\circ}$$

Obciążenie indukcyjne

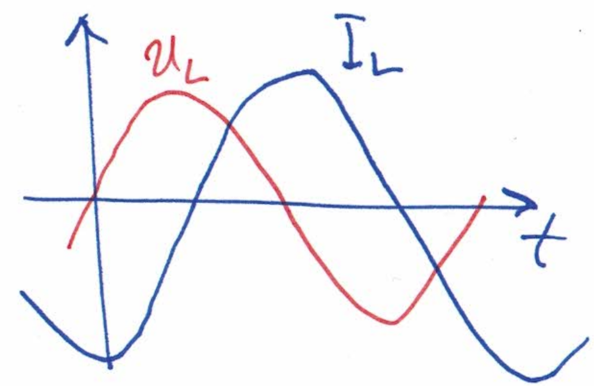
$$U_L = U_{L\,max} \sin \omega_w t$$



$$U_L = L \frac{dI_L}{dt}$$

$$\frac{dI_L}{dt} = \frac{U_{L\,max}}{L} \sin \omega_w t$$

$$I_L = -\frac{U_{L\,max}}{\omega_w L} \cos \omega_w t$$



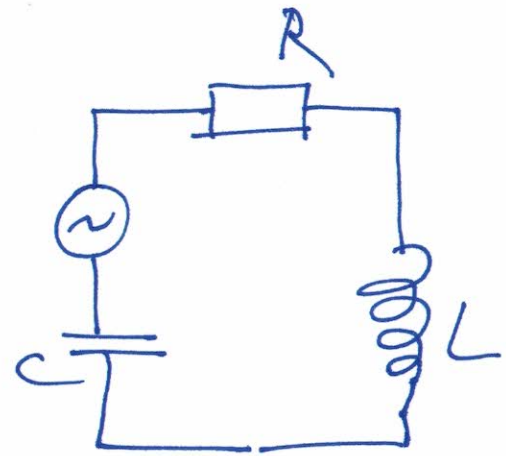
Reaktancja indukcyjna cewki: $X_L = \omega_w L$

$$\boxed{I_L = \frac{U_{L\,max}}{X_L} \sin(\omega_w t - \phi)}$$

$$\boxed{\phi = 90^\circ}$$

$$\boxed{U_{L\,max} = I_{L\,max} X_L}$$

Obwód RLC



$$\mathcal{E} = \mathcal{E}_{max} \sin \omega_w t$$

$$I = I_{max} \sin(\omega_w t - \phi)$$

$$I_{max} = \frac{\mathcal{E}_{max}}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$ – impedancja

$$I_{max} = \frac{\mathcal{E}_{max}}{\sqrt{R + \left(\omega_w L - \frac{1}{\omega_w C}\right)}}$$

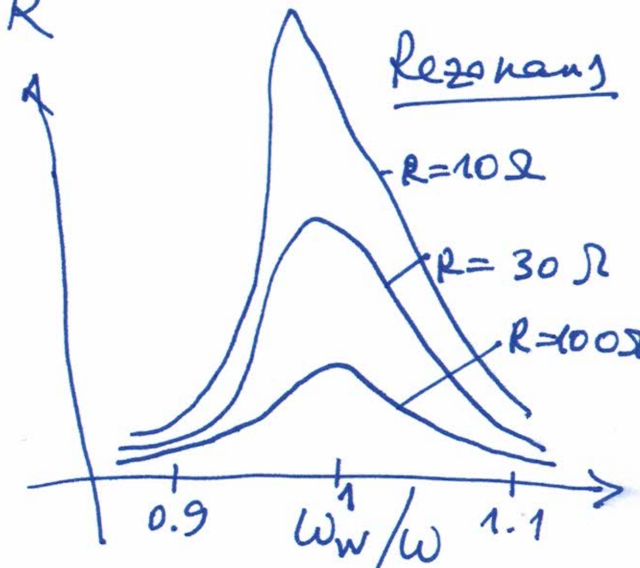
$$\operatorname{tg} \phi = \frac{X_L - X_C}{R}$$

Jeśli $X_L = X_C$ $\quad \phi = 0$, $\quad I_{max} = \frac{\mathcal{E}_{max}}{R}$

$$\boxed{\omega = \omega_w = \frac{1}{\sqrt{LC}}}$$

$$\mathcal{E} = \mathcal{E}_{max} \sin \omega_w t$$

Rezonans



Moc w obwodach prądu zmiennego

$$P = I^2 R = I_{max}^2 R \sin^2(\omega_w t - \phi)$$ – szybkość rozpraszania energii na oporniku

Średnia szybkość rozpraszania energii

$$P_{śr} = \frac{I_{max}^2 R}{2} \Rightarrow \boxed{I_{sk} = \frac{I_{max}}{\sqrt{2}}}$$ skuteczne wartość natężenia prądu

$$P_{śr} = I_{sk}^2 R$$

Zdefiniujemy skuteczne napięcie: $U_{sk} = \frac{U_{max}}{\sqrt{2}}$ i $\mathcal{E}_{sk} = \frac{\mathcal{E}_{max}}{\sqrt{2}}$ (SEM)

$$I_{sk} = \frac{\mathcal{E}_{sk}}{Z} = \frac{\mathcal{E}_{sk}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

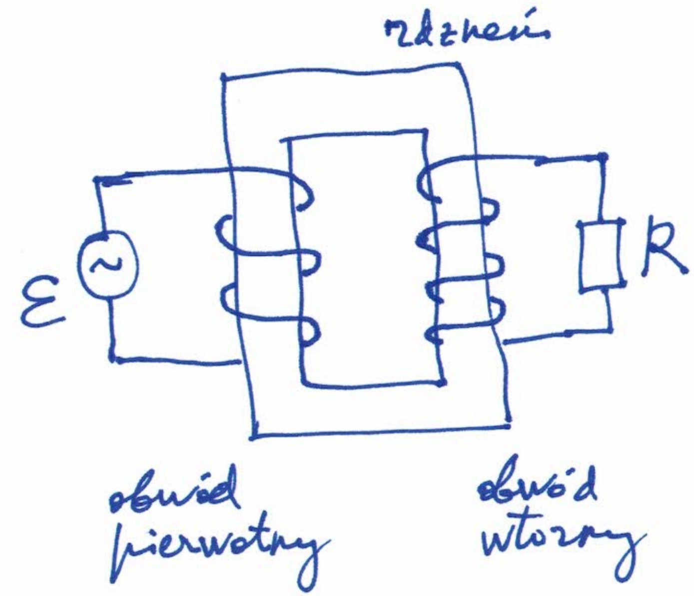
$$P_{śr} = \frac{\mathcal{E}_{sk}}{Z} I_{sk} R = \mathcal{E}_{sk} I_{sk} \frac{R}{Z} = \mathcal{E}_{sk} I_{sk} \cos\phi \,, \quad \frac{R}{Z} = \underbrace{\cos\phi}_{\text{współczynnik mocy}}$$

$$\boxed{P_{śr} = \mathcal{E}_{sk} I_{sk} \cos\phi}$$

Transformatory

$$\mathcal{E} = \mathcal{E}_{max} \sin \omega t$$

SEM, indukowana w obwodzie wtórnym

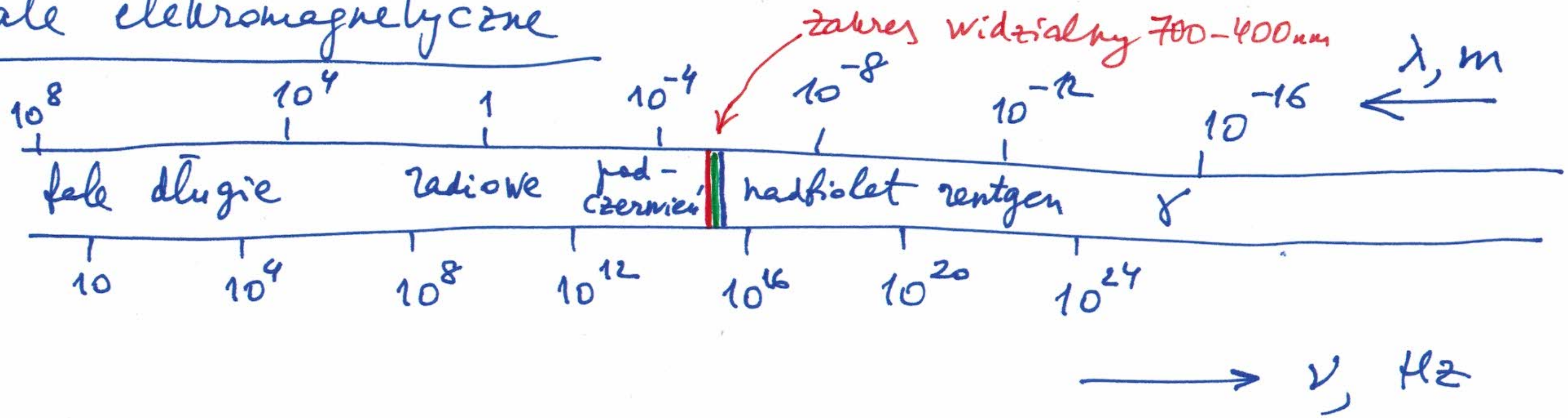


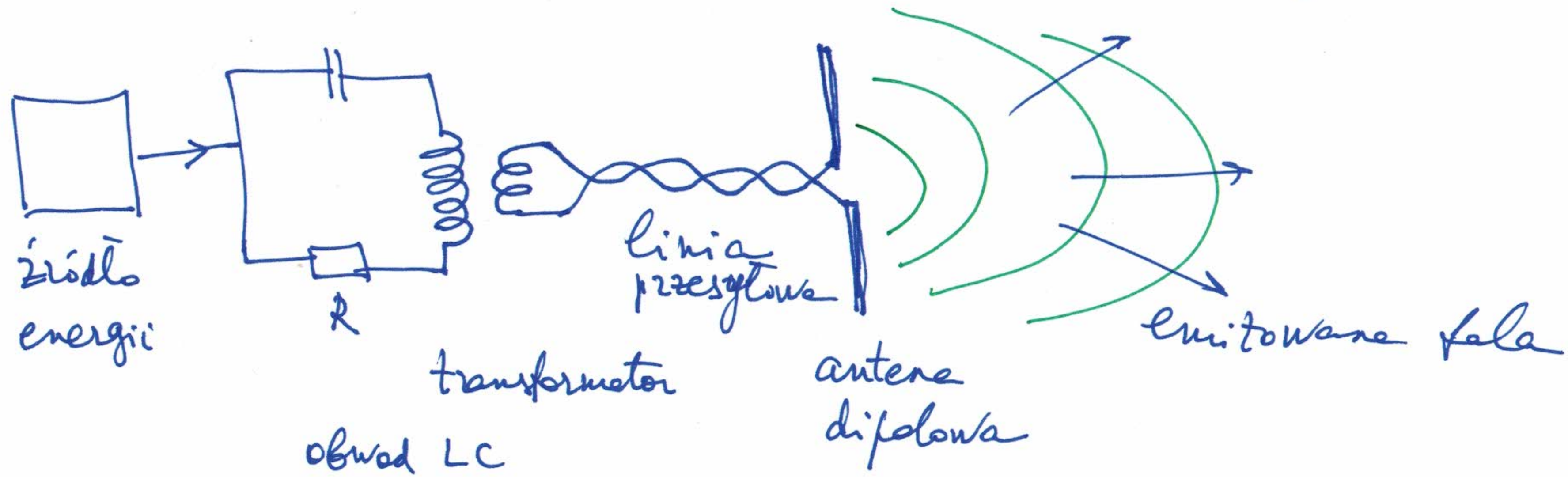
$$\mathcal{E}_z = \frac{d\Phi_B}{dt} = \frac{U_p}{N_p} = \frac{U_W}{N_W}$$

$$\boxed{U_W = U_p \frac{N_W}{N_p}}$$ transformacja napięcia

$$I_W = I_p \frac{N_p}{N_W}$$ – transformacja prądów

Fale elektromagnetyczne





Pole elektryczne i magnetyczne w fali

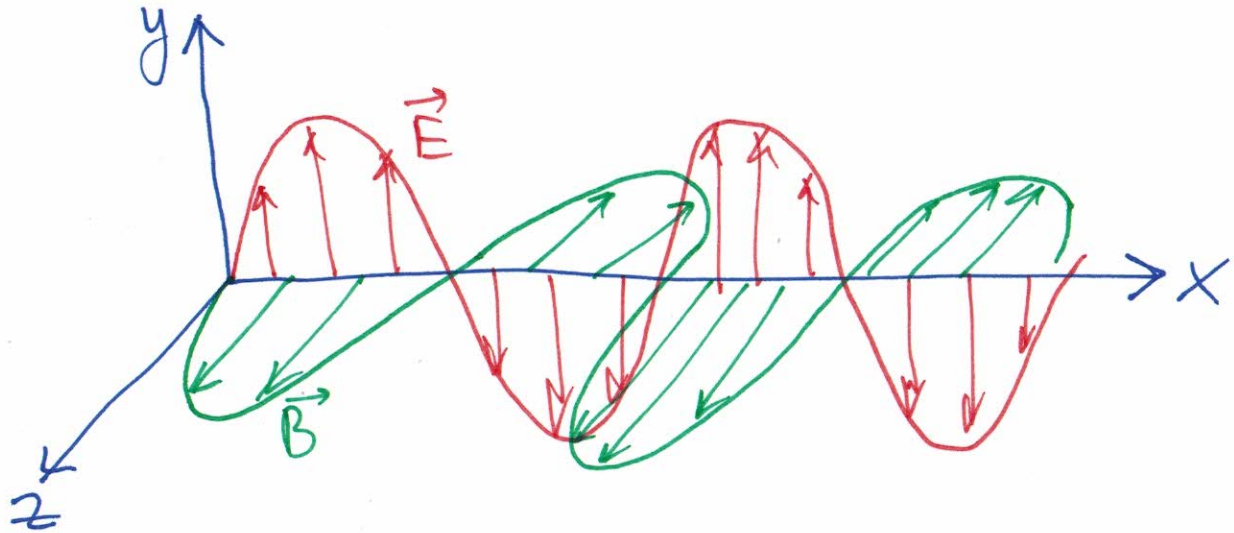
$$\boxed{\begin{aligned} E &= E_m \sin(kx - \omega t) \\ B &= B_m \sin(kx - \omega t) \end{aligned}}$$

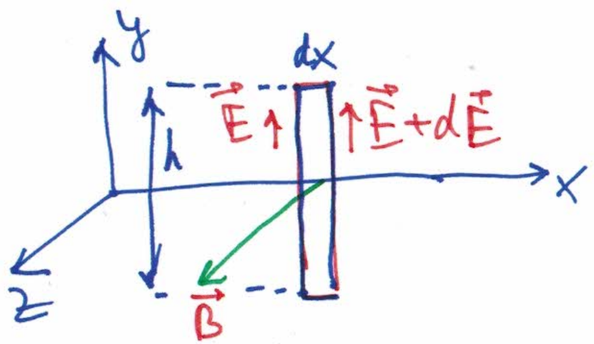
$$\frac{E_m}{B_m} = c$$

Prędkość rozchodzenia się fali:

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \simeq 3 \cdot 10^8 \frac{m}{s}}$$

Fale elektromagnetyczne





Prawo indukcji Faradaya

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$(E+dE)h - Eh = -h\,dx\,\frac{dB}{dt} \qquad \Phi_B = B\,h\,dx$$

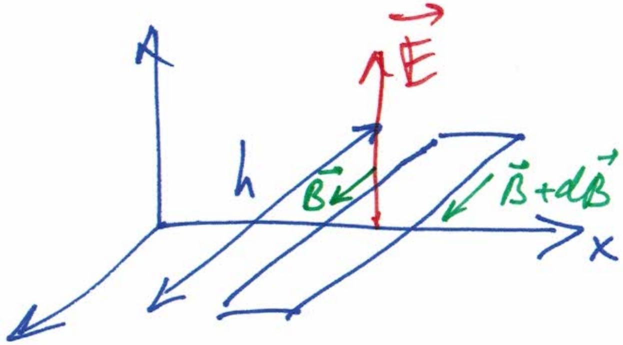
$$h\,dE = -h\,dx\,\frac{dB}{dt}$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

$$E = E_m \sin(kx-\omega t)$$

$$B = B_m \sin(kx-\omega t)$$

$$\boxed{\frac{E_m}{B_m} = c} \qquad c = \frac{\omega}{k}$$



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\Phi_E = E h\, dx$$

$$-(B+dB)h + Bh = \underbrace{-h\,dB = h\,dx \frac{dE}{dt}}\cdot \mu_0 \varepsilon_0$$

$$-h\,dB = \mu_0 \varepsilon_0\, h\, dx \frac{dE}{dt}$$

$$-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0 \varepsilon_0 c} \quad \Rightarrow \quad \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

Przepływ energii i wektor Poyntinga

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$ wektor Poyntinga

Jednostki:

$$S = \frac{\text{energia/czas}}{\text{pole powierzchni}} = \frac{\text{moc}}{\text{pole powierzchni}}$$

Kierunek wektora Poyntinga $\vec{S}$ fali elektromagnetycznej jest kierunkiem rozchodzenia się fali i przepływu energii

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$ – chwilowa szybkość przepływu energii